List of projects

1 Homology with coefficients and the universal coefficients theorem for homology

- Given an abelian group G define the homology groups $H_n(X; G)$ of a topological space X with coefficents in G.
- Prove the universal coefficients theorem that relates homology and homology with coefficients.

2 Eilenberg–Steenrod axioms. Uniqueness of singular homology

- State the Eilenberg–Steenrod axioms for unreduced homology
- Prove that if h_* is any unreduced homology theory and $h_n(*) = \mathbb{Z}$ for $n = 0$ and 0 otherwise, then $h_n \cong H_n$, where H_n denotes singular cohomology.

3 Homology of product spaces. Künneth theorem

- Define the tensor product of chain complexes. State the Eilenberg–Zilberg theorem
- Prove the Künneth theorem that relates $H_n(X \times Y)$ to $H_n(X)$ and $H_n(Y)$.

4 Lens spaces and their homology

- Describe the lens space $L(p, q)$ for p and q relatively prime positive integers.
- Compute $H_n(L(p, q))$ for all $n \geq 0$.

5 Simplicial homology

- Define the simplicial homology groups of an (abstract) simplicial complex.
- Prove that the simplicial homology groups of a simplicial complex are isomorphic to the singular homology groups of its realization.

6 Cohomology and the universal coefficient theorem

- Define the cohomology groups of a topological space.
- Prove the universal coefficients theorem that relates homology and cohomology.

7 The Hurewicz theorem

- Define the Hurewicz homomorphism $h_n\colon \pi_n(X) \to H_n(X)$.
- Prove that, for $n \geq 2$, if X is $(n-1)$-connected, then $H_k(X) = 0$ for $k < n$ and h_n is an isomorphism.

8 Classification of surfaces and their homology

- State the classification theorem for compact surfaces.
- Compute the homology groups of compact surfaces.

References

[1] W. Fulton, *Algebraic Topology: a First Course*, GTM 153, Springer Verlag, 1995

[2] A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.

[3] W. S. Massey, *Singular Homology Theory*, GTM 70, Springer-Verlag, 1991.

[4] W. S. Massey, *A Basic Course in Algebraic Topology*, GTM 127, Springer-Verlag, 1997.

[5] J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing Company, 1984.

[6] J. J. Rotman, *An Introduction to Algebraic Topopogy*, Springer-Verlag, 1998.

[7] E. H. Spanier, *Algebraic Topopogy*, Springer-Verlag, 1966.

[8] J. W. Vick, *Homology Theory*, GTM 145, Springer-Verlag, 1994.